

Constraining CSL strength parameter λ from standard cosmology and spectral distortions of CMBR

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Abstract

Models of spontaneous wave function collapse modify the linear Schrödinger equation of standard Quantum Mechanics by adding stochastic non-linear terms to it. The aim of such models is to describe the quantum (linear) nature of microsystems along with the classical nature (violation of superposition principle) of macroscopic ones. The addition of such non-linear terms in the Schrödinger equation leads to non-conservation of energy of the system under consideration. Thus, a striking feature of collapse models is to heat non-relativistic particles with a constant rate. If such a process is physical, then it has the ability to perturb the well-understood thermal history of the universe. In this article we will try to investigate the impacts of such heating terms, according to the Continuous Spontaneous Localization (CSL) model, on standard evolution of non-relativistic matter and on the formation of CMBR. We will also put constraints on the CSL collapse rate λ by considering that the standard evolution of non-relativistic matter is not hampered and the observed precise blackbody spectrum of CMBR would not get distorted (in the form of μ -type and y -type distortions) so as to violate the observed bounds.

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1. INTRODUCTION

Models of spontaneous wave function collapse [1, 2] aim to unify the dynamics of microscopic and macroscopic systems in order to answer the long prevailing question of Quantum “Measurement problem”. The unification of microscopic (superposition of states) and macroscopic (violation of superposition principle) dynamics is accomplished by modifying the Schrödinger equation through adding non-linear stochastic terms. The non-linear terms in the modified Schrödinger equation ensures the breakdown of superposition principle at the macroscopic level and the stochastic nature of such dynamics indicates that the outcome of measurements would be probabilistic. The added non-linear terms act as amplification mechanism to ensure that these modifications have negligible impacts on microscopic system but are very efficient of localization for macroscopic ones. It is also important to note that the added stochastic terms also respect causality of the dynamics as deterministic non-linear evolution of Schrödinger equation leads to violation of relativity as shown in [3]. Among the many attempts of constructing such a modified dynamics, Quantum Mechanics of Spontaneous Localization model (QMSL, later known as GRW model after the name of the authors [4]), Quantum Mechanics with Universal Position Localization model (QMUPL [5, 6]) and Continuous Spontaneous Localization (CSL [7, 8]) model are worth noting. In some models it is considered that localization of wave-packets is a consequence of gravitational effects [9–11]. In this article we will concentrate only on CSL model and analyze the signatures of such a model while applying it in cosmology.

According to the present understanding of the CSL model, a theoretical origin of the free parameters introduced in the scheme, i.e. the collapse rate λ and the width of the localization r_c , are yet to be determined. Hence values of such free parameters have to be obtained phenomenologically. Several proposed bounds on the free parameter λ (considering $r_c \approx 10^{-5}$ cm) can be found in [2, 12].

One distinguishing feature of the CSL model, common to most collapse models, is that due to the presence of non-linear stochastic terms in the Schrödinger equation the total energy of the system does not remain constant and the non-relativistic massive particles within a system gain energy with a constant rate proportional to the CSL parameter λ . Such heating effects are interesting to study for systems which are generally in thermal equilibrium. Study of atomic and nuclear systems [8] which demands the non-dissociation

of cosmic hydrogen by CSL heating obtains an upper bound as $\lambda < 1 \text{ s}^{-1}$, whereas studies of proton decay considering CSL models [13] also lead to similar bounds. Comparing with the experimental bounds on photon emission from Germanium one can constrain the CSL parameter as $\lambda < 10^{-11} \text{ s}^{-1}$. Furthermore, in a cosmological scenario heating of protons through CSL mechanism over universe's life-time [1, 4, 13] and the rise of CMBR temperature due to interactions of CMBR photons with these heated protons [13] have been studied to suggest a bound as $\lambda < 10^{-5} \text{ s}^{-1}$. Also thermal equilibrium of Inter Galactic Medium (IGM) [13] has been studied to put an upper bound as $\lambda < 10^{-8} \text{ s}^{-1}$.

However, as the thermal evolution of our universe is well studied and constrained observationally, there can be other cosmological scenarios, apart from the evolution of IGM, where the imprints of the anomalous heating of CSL model can be studied. One such scenario, which we will exploit in the present article, is the thermodynamic equilibrium of matter and radiation before the formation of CMBR. Injection of energy during such epochs perturbs the thermodynamic equilibrium of matter and radiation in the cosmic plasma which leads to distortions in the well-measured blackbody spectrum of CMBR. Such distortions are stringently constrained by observations. Here we will address two types of distortions of CMBR : the μ -distortion, which results from energy releases during the redshift span $2 \times 10^6 > z > 5 \times 10^4$, and the y -distortion, which comes from energy releases during the redshift span $1100 < z < 5 \times 10^4$. μ -distortion yields a non-vanishing, frequency-dependent small chemical potential of the cosmic photons which leads to a Bose-Einstein distribution in the high-frequency regime of the spectrum rather than the pure Planckian spectrum of a blackbody [14]. Observations of COBE/FIRAS pointed out that such a distortion should be very small and puts an upper bound on such distortions as $\mu \lesssim 9 \times 10^{-5}$ with 95% confidence [15], whereas an upcoming experiment PIXIE can probe such distortions up to $\mu \sim 5 \times 10^{-8}$ [16]. On the other hand, the y -distortion of the spectrum is characterized by the lower density of photons in the low-frequency regime and increment of photon number in the high-frequencies with respect to a standard Planckian blackbody spectrum [17]. COBE/FIRAS observations put an upper bound on observed y -parameter as $y \leq 1.5 \times 10^{-5}$ with 95% confidence [15], whereas PIXIE can put an upper bound on such distortion as $y \leq 10^{-8}$ [16].

In this present article we will explore the possibilities of generating spectral distortions of CMBR due to CSL heating of non-relativistic particles. Before any such endeavor, one

should confirm that evolution of non-relativistic matter during the radiation and matter dominated era should not get affected by such anomalous heating of CSL and they evolve according to the standard cosmology. Then we note here that as such an anomalous CSL heating of non-relativistic matter can disturb the thermodynamic equilibrium of matter and radiation before CMBR formation, it can lead to spectral distortions of the CMBR where the amount of distortions will depend upon the strength of the heating of the particles. Thus by quantifying the amount of distortions such CSL heating can generate, one can put bounds on the free parameter λ of the theory.

However, in the present article we will not consider the effects of CSL heating on the evolution of relativistic matter including radiation. It has been a challenge to consistently formulate any relativistic generalization of the collapse models such as CSL and various attempts have been made in that direction. For a brief review of the attempts made to make a relativistic generalization of Spontaneous Collapse models one may refer to [2]. Thus, here we will assume that the relativistic matter evolve according to the standard cosmology. Any properly developed relativistic collapse model can, in principle, leave its imprint on CMBR and will be worth studying in future.

According to standard Friedmann-Robertson-Walker (FRW) cosmology at $z_i \approx 2 \times 10^6$ the temperature of the cosmic soup is $T_i \approx 470$ eV which is much lower than the rest mass energy of electrons which is 0.5 MeV. Thus the electrons and protons present in the cosmic soup during the concerned epochs i.e. $2 \times 10^6 > z > 1100$ can be treated as non-relativistic particles. Furthermore, if one tracks the density profile $n(z) = n_0(1+z)^3$, where n_0 is the present number density of baryons today, then one obtains the number of baryons per unit correlation cell of size $r_c^3 \sim 10^{-21} \text{ m}^3$ (prescribed in CSL model) to be smaller than unity at $z_i \approx 10^6$ which falls off even further with the expansion of the universe. This indicates that considering the electrons and protons as free particles during such epochs is a good approximation. Hence the above discussion shows that during such epochs CSL model is applicable considering the massive particles in the cosmic soup to be free and non-relativistic. This enforces one to seek for possible spectral distortions in the CMBR generated due to CSL heating and to put bounds on λ by quantifying the amount of distortions that can be yielded by such heating.

We have organized the present article in the following manner. In Sec. (2) we will briefly review the CSL model and estimate the rate of heating of non-relativistic particles due

to presence of non-linear stochastic terms in the Schrödinger equation in the CSL scenario. Here, we will discuss two scenarios of CSL model : firstly the generic case where the strength parameter λ is independent of mass and a variant scenario where λ is dependent on the mass of the non-relativistic particles under consideration. We have mentioned before that one should ensure that CSL heating would not disturb the standard evolution of the non-relativistic matter during radiation and matter dominated era. Sec. (3) is devoted to such discussions and to constrain the strength parameter λ by demanding the standard cosmological evolution of non-relativistic matter throughout. In Sec. (4) we will first give a brief review of μ -distortion and then we will put bounds on both the scenarios of CSL model discussed above. Sec. (5) is focused to discuss the y -distortion that CSL heating of non-relativistic particles can generate and constraining the CSL parameter from such distortion of CMBR spectrum. In the penultimate section (Sec. (6)) we will briefly consider the case where spontaneous photon emission is taken into account within the arena of CSL model and show that such feeble emission of photon in the high-frequency regime would be insufficient to generate any considerable distortion in the CMBR spectrum and thus would not be an important feature to constrain the strength parameter λ . In the last section we will discuss all the bounds we will obtain throughout and then conclude.

2. REVIEWING CSL MODEL AND EXPLAINING THE HEATING OF PARTICLES

In this section we will briefly review the CSL mechanism and how it leads to heating of non-relativistic particles with a constant rate. We will initially describe the GRW model of spontaneous collapse [4] where randomly acting non-linear terms are added to the Schrödinger equation, based on the assumption that the constituents of a system suddenly collapse to a localized wave-function with an appropriate rate. Concepts of such models are easy to conceive and the main result for energy gain by the non-relativistic particles are the same in both GRW and CSL models which makes the GRW model worth describing at this point. In literature, at later times GRW model has been extended to CSL model by appropriately introducing stochastic terms in the Schrödinger equation combined with the theory of continuous Markov processes in Hilbert space. We will discuss the CSL model in the later subsection.

2.1. GRW model of spontaneous wave function collapse

A deviant of quantum mechanics, called the GRW model [4], allows for spontaneous localization of a particle with a mean rate λ , where the evolution of the system between two successive localizations is according to the standard Schrödinger equation. According to this theory a localization operator,

$$L_{\mathbf{x}}^i = \left(\frac{\alpha}{\pi}\right)^{\frac{3}{4}} e^{-\frac{\alpha}{2}(\mathbf{q}_i - \mathbf{x})^2}, \quad (1)$$

localizes a wave-function $|\psi\rangle$ in space (at point \mathbf{x}) yielding $|\psi_{\mathbf{x}}^i\rangle/||\psi_{\mathbf{x}}^i\rangle||$, where $|\psi_{\mathbf{x}}^i\rangle \equiv L_{\mathbf{x}}^i|\psi\rangle$ is the wave-function of the i^{th} particle localized at \mathbf{x} . Here $1/\sqrt{\alpha} \equiv r_c$ quantifies the accuracy of the localization at position \mathbf{q}_i . Due to the stochasticity of the spontaneous localization, a system in a pure state, which consists of a single particle, is transformed into a statistical mixture :

$$|\psi\rangle\langle\psi| \rightarrow \int d^3x L_{\mathbf{x}}^i |\psi\rangle\langle\psi| L_{\mathbf{x}}^i \equiv T[|\psi\rangle\langle\psi|]. \quad (2)$$

Analogously, if the system is initially in a statistical mixture, given by the operator ρ , then the localization of the system is determined by $T[\rho]$ with a probability λ . Hence the statistical state ρ after time-interval dt can be obtained as

$$\rho(t + dt) = (1 - \lambda dt) \left[\rho(t) - \frac{i}{\hbar} [H, \rho(t)] dt \right] + \lambda dt T[\rho(t)], \quad (3)$$

where the first term on R.H.S. indicates that the system evolves according to Schrödinger equation if it is not localized within the interval dt . Thus the evolution of the system, represented by operator ρ , will be according to the *master equation* :

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar} [H, \rho(t)] - \lambda (\rho(t) - T[\rho(t)]). \quad (4)$$

A simple example of free particle Hamiltonian in one dimension has been considered in [1, 4] where the solution of the master equation in coordinate space has been obtained as

$$\langle q' | \rho(t) | q'' \rangle = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dk \int_{-\infty}^{+\infty} dy e^{-\frac{i}{\hbar}ky} F(k, q' - q'', t) \langle q' + y | \rho_S(t) | q'' + y \rangle, \quad (5)$$

where $\langle q' | \rho_S(t) | q'' \rangle$ is the solution of the pure Schrödinger equation and the factor

$$F(q, k, t) = e^{-\lambda t + \lambda \int_0^t d\tau e^{-\frac{\alpha}{4}(q - k\tau/m)^2}} \quad (6)$$

encapsulates all the dynamics of localization. $\lambda \rightarrow 0$ yields $F(q, k, t) = 1$ which shows in this limit $\langle q' | \rho_S(t) | q'' \rangle = \langle q' | \rho(t) | q'' \rangle$, as expected. The dynamical evolution of this free particle system yields a spread in the momentum as [4]

$$\langle \hat{p}^2 \rangle \equiv \text{tr}[\hat{p}^2 \rho(t)] = \langle \hat{p}^2 \rangle_S + \frac{3\alpha\lambda\hbar^2}{2}t, \quad (7)$$

which can be derived using Eq. (4). Here, $\langle \hat{p}^2 \rangle_S$ is the conserved momentum for the free Schrödinger evolution. Thus for a non-relativistic particle with energy $E = \frac{p^2}{2m}$ the spread in energy will be

$$\langle E \rangle = \langle E \rangle_S + \frac{3\alpha\lambda\hbar^2}{4m}t, \quad (8)$$

which shows non-conservation of energy of the system with time. Therefore, one can infer that due to spontaneous localization of particles in a system a non-relativistic particle gains energy with a rate

$$\frac{\partial E}{\partial t} = \frac{3\alpha\lambda\hbar^2}{4m}. \quad (9)$$

This heating rate has been obtained for one non-relativistic particle. To investigate effects of such spontaneous localizations in cosmology one has to obtain the total heating rate for the cosmic plasma containing non-relativistic electrons and protons.

2.2. Continuous Spontaneous Localization (CSL) model

Out of many spontaneous collapse models, the one that is commonly used in physical applications is the Continuous Spontaneous Localization (CSL) model which generalizes the original GRW model [4] to systems of identical particles. In the CSL model the collapse of the wave-function happens continuously in time in contrast to the GRW model where the collapse of wave-function happens discretely. The parameters introduced in this model, in a similar way as had been done in GRW model, are the coupling constant γ and the correlation length r_c (which is conventionally taken as 10^{-5} cm). The modified Schrödinger equation takes the form in CSL model [1]

$$|d\psi_t\rangle = \left[-\frac{i}{\hbar}Hdt + \sqrt{\gamma} \int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle_t) dW_t(\mathbf{x}) - \frac{\gamma}{2} \int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle_t)^2 dt \right] |\psi_t\rangle. \quad (10)$$

Here the operator $N(\mathbf{x})$ is an averaged density operator given as

$$N(\mathbf{x}) = \sum_s \int d^3y g(\mathbf{y} - \mathbf{x}) a_s^\dagger(\mathbf{y}, s) a(\mathbf{y}, s), \quad (11)$$

where the sum is over various particle species s , with $a_s^\dagger(\mathbf{y}) a_s(\mathbf{y})$ being the number density operator of species s and $W_t(\mathbf{x})$ is the family of standard Wiener process for each point in space. Moreover, $g(\mathbf{x})$ is a spherically symmetric, positive, real function peaked around $\mathbf{x} = 0$ with the normalization $\int d^3x g(\mathbf{x}) = 1$ and can be written as

$$g(\mathbf{x}) = \frac{1}{(2\pi r_c^2)^{3/2}} e^{-\mathbf{x}^2/r_c^2}. \quad (12)$$

The collapse rate λ of GRW model is related to that of the CSL parameter γ as

$$\lambda = \frac{\gamma}{8\pi^{3/2} r_c^3}. \quad (13)$$

The corresponding master equation in CSL model as for Eq. (4) in GRW is given by [1]:

$$\frac{d\rho(t)}{dt} = \frac{i}{\hbar} [H, \rho(t)] + \gamma \int d^3x N(\mathbf{x}) \rho(t) N(\mathbf{x}) - \frac{\gamma}{2} \int d^3x \{N^2(\mathbf{x}), \rho(t)\}. \quad (14)$$

It is to be noted from this relation that λ of GRW model is directly proportional to the parameter γ of CSL model apart from some constant factors. In such a case a bound on λ will directly indicate a bound on γ . Hence we will consider bounds in λ for further discussions and those can be converted into bounds on γ .

In a generic CSL model scenario the introduced parameter γ (or, equivalently, λ) is independent of the mass of the constituent particles. In literature, a variant of such a model is also discussed where the parameter λ is mass-dependent as

$$\lambda(m) = \lambda_0 \left(\frac{m}{m_N} \right)^\beta, \quad (15)$$

where m_N is the mass of a nucleon. Such a model is motivated from the feature that in this scenario the collapse rate is different for different species bearing different masses [8]. We provide a justification for such a choice. The calculations of the reduction rate of wave-functions in [1] shows that the off-diagonal elements of coordinate space density matrix for a single nucleon approach zero exponentially with a reduction rate Γ_R given by

$$\Gamma_R = \lambda \left(1 - e^{-\mathbf{x}^2/4r_c^2} \right), \quad (16)$$

which for $|\mathbf{x}| > r_c$ becomes

$$\Gamma_R \approx \lambda. \quad (17)$$

For n nucleons within a radius smaller than the correlation length, in a superposition of states with distance larger than r_c , this rate is multiplied by n^2 to yield

$$\Gamma_R \simeq n^2 \lambda. \quad (18)$$

In mass-proportional CSL model one wants the collapse rate of a massive single particle composed of n fundamental particles to be the same as the collapse rate of n single fundamental particles. This is true when the strength parameter λ is quadratically proportional to the mass of the constituent particle i.e. $\beta = 2$. In that case the reduction rate of the system of n fundamental particles becomes

$$\Gamma_R = \lambda_0 n^2 \left(\frac{m}{m_N} \right)^2. \quad (19)$$

Thus in this article, along with the mass-independent CSL model with strength parameter λ , we will also analyze the mass-dependent CSL model where the strength parameter λ_0 is for the quadratically mass-dependent case and compare the bounds obtained in both the cases.

It has been shown in [8] that due to the presence of non-linear terms in the Schrödinger equation in the CSL model the energy will not be conserved (as has also been discussed for GRW model). Since, as discussed in the introduction, the number of baryons in the universe per unit correlation cell of size $r_c^3 \sim 10^{-21} \text{ m}^3$ is smaller than unity, for the considered times, the effect of the identity of particles can be neglected and the rate energy increase in such a scenario is similar to that in the GRW model which is given in Eq. (9). For CSL model the non-conservation of energy is quantified as [8]

$$\langle E \rangle = \frac{3\lambda\alpha\hbar^2}{4m}t. \quad (20)$$

Such a heating rate has been obtained for one non-relativistic particle. To investigate effects of such spontaneous localization in cosmology one has to obtain the total heating rate of the cosmic plasma containing non-relativistic electrons and protons. Given the number density n_s for each species ($s = e, p$) the total energy density gain by each species can be straightforwardly obtained from the above equation as

$$\frac{\partial \varepsilon_s}{\partial t} = \frac{3\lambda\alpha\hbar^2}{4m_s}n_s, \quad (21)$$

taking into account that during the epochs one is interested in, these particles are non-relativistic and behave as free particles in the cosmic plasma.

While applying a generic CSL model in cosmology, where the parameter introduced in the theory is mass-independent, the contribution to the total change in energy density will come from the electron fluid as $\frac{1}{m_e} \gg \frac{1}{m_p}$ and $n_e \approx n_p$. Thus in such a scenario the change in total energy can be written using the above equation as

$$\frac{\partial \varepsilon}{\partial t} = \frac{3\alpha\lambda\hbar^2}{4m_e} n_e. \quad (22)$$

On the other hand, if one considers the variant scenario of CSL model where the parameter λ is dependent on mass of the particle quadratically, then the proton fluid will contribute more to the total change in the energy density and in such a case one can write

$$\frac{\partial \varepsilon}{\partial t} = \frac{3\alpha\lambda_0\hbar^2}{4m_p} n_p, \quad (23)$$

where the mass of a nucleon can be taken as that of a proton $m_N \approx m_p$.

It is worth pointing out at this point that the origin of the stochastic field $W_t(\mathbf{x})$ in the non-linear Schrödinger equation of CSL model is not yet known and thus it is difficult to point out at this moment how such a stochastic field can be cosmologically accounted for. In this work, we will thus not consider this stochastic field at all but only analyze the energy-increase of standard non-relativistic particles while interacting with such fields.

3. BOUNDS ON CSL PARAMETER FROM RADIATION DOMINATED AND MATTER DOMINATED ERA

Before investigating the spectral distortions in CMBR caused by CSL heating of non-relativistic particles, it is important to ensure that such heating does not hamper the standard evolution history of the universe. We can only consider those epochs where the particles can be treated as non-relativistic particles and hence allow one to apply the methods of CSL model.

In standard FRW cosmology the scale factor a is related to the corresponding redshift z as $1 + z = \frac{a_0}{a}$ where a_0 is the present scale factor. This implies that $\frac{da}{a} = -\frac{dz}{1+z}$. It is also considered that the universe evolves adiabatically which implies that the comoving entropy is always conserved. For such isentropic processes $a^3 T^3$ remains constant in a comoving

volume which yields $T \propto \frac{1}{a} \propto (1+z)$ i.e. $T(z) = T_0(1+z)$ where $T_0 = 2.73$ K is the present temperature of the CMBR. Writing the temperature in eV units one has

$$T(z) \approx 2.4 \times 10^{-4}(1+z) \text{ eV}, \quad (24)$$

which implies that when the temperature of the universe is 0.5 MeV (i.e. of the order of the rest mass energy of the electrons) the corresponding redshift would be 2×10^9 . Thus for radiation dominated (RD) universe we would be interested in the redshift span of $2 \times 10^9 < z < 3233$ and for that of matter dominated (MD) era would be $3233 < z < 1$ (where $z \approx 3233$ is the epoch of matter-radiation equality).

Due to CSL heating the energy density of matter will evolve differently than standard cosmology. According to standard cosmology the energy density of matter gets diluted with the expansion of the universe as

$$\frac{d\rho_M(z)}{dt} = -3H(z)\rho_M(z), \quad (25)$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter and we have taken the pressure $p = 0$ for matter (non-relativistic particles). But the change in the matter energy density with time ($\frac{d\rho_M}{dt}$) in this case is affected by both the expansion of the universe and the CSL heating which is proportional to the matter number density present during that time in the cosmic soup. Then one can write the evolution of the matter density using Eq. (21), in the case where radiation and matter are not tightly coupled, as

$$\frac{d\rho_M(z)}{dt} = -3H(z)\rho_M(z) + \sum_{s=e,p} \frac{3\lambda\hbar^2\alpha}{4m_s} n_M(z). \quad (26)$$

In standard cosmology the energy density ρ_M of non-relativistic electrons and protons can be written in terms of their number density n_M as

$$\rho_M = n_M(m_e + m_p)c^2, \quad (27)$$

where the number density of electron n_e and that of proton n_p are the same $n_e \approx n_p \approx n_M$.

Using this in Eq. (26) one can write as

$$\frac{d\rho_M(z)}{\rho_M(z)} = -3\frac{da}{a} + \sum_{s=e,p} \frac{3\lambda\hbar^2\alpha}{4m_s(m_e + m_p)c^2} dt. \quad (28)$$

With the definition of Hubble parameter H and the relation $1+z = \frac{a_0}{a}$ it can be seen that

$$dt = -\frac{dz}{(1+z)H(z)} \quad (29)$$

and along with the relation $\frac{da}{a} = -\frac{dz}{1+z}$ the evolution equation of energy density can be written as

$$\frac{d\rho_M(z)}{\rho_M(z)} = 3\frac{dz}{1+z} - \sum_{s=e,p} \frac{3\lambda\hbar^2\alpha}{4m_s(m_e + m_p)c^2} \frac{dz}{(1+z)H(z)}. \quad (30)$$

The Hubble parameter $H(z) = \frac{\dot{a}}{a}$ can be written in terms of dimensionless quantities using the Friedmann equation as

$$H(z) = H_0 \sqrt{\Omega_{R_0}(1+z)^4 + \Omega_{M_0}(1+z)^3 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}, \quad (31)$$

where $\Omega_i \equiv \rho_i/(3m_p^2 H^2)$ is called the density parameter for the matter species i and Ω_{R_0} , Ω_{M_0} , Ω_{k_0} and Ω_{Λ_0} are the present radiation, matter, curvature and dark energy density parameter respectively. During RD era, the radiation fluid contributes dominantly to the total density and thus the Hubble parameter can be written as

$$H(z) = H_0 \Omega_{R_0}^{\frac{1}{2}} (1+z)^2, \quad (32)$$

whereas during MD era that would be

$$H(z) = H_0 \Omega_{M_0}^{\frac{1}{2}} (1+z)^{\frac{3}{2}}. \quad (33)$$

Let us consider the RD era first. It is worth noting at this point that during radiation era matter and radiation will be tightly coupled and during this period Compton scattering is much more efficient than CSL heating (as will be argued below). Also as the ratio of entropy in baryons and photons is 10^{-9} the baryons will lose most of its energy, gained by CSL heating, to photons via Compton scattering retaining only a fraction f of the order 10^{-9} . Keeping this in mind and using Eq. (32) in Eq. (30) one gets

$$\frac{d\rho_M(z)}{\rho_M(z)} = 3\frac{dz}{1+z} - \sum_{s=e,p} \frac{3f\lambda\hbar^2\alpha}{4m_s(m_e + m_p)c^2 H_0 \Omega_{R_0}^{\frac{1}{2}}} \frac{dz}{(1+z)^3}, \quad (34)$$

solving which one gets

$$\rho_{M_f} = \exp \left\{ K_{\text{CSL,RD}} \left[\frac{1}{(1+z_f)^2} - \frac{1}{(1+z_i)^2} \right] \right\} \left(\frac{1+z_f}{1+z_i} \right)^3 \rho_{M_i}, \quad (35)$$

where we have defined

$$K_{\text{CSL,RD}} \equiv \sum_{s=e,p} \frac{3 \times 10^{-9} \lambda \hbar^2 \alpha}{8m_s(m_e + m_p)c^2 \Omega_{R_0}^{\frac{1}{2}} H_0}. \quad (36)$$

It is evident from the above equation that in absence of CSL heating (i.e. with $\lambda = 0$) we have the standard cosmological evolution of matter density as

$$\rho_{M_f} = \left(\frac{1 + z_f}{1 + z_i} \right)^3 \rho_{M_i}. \quad (37)$$

It is important to note that before recombination the free electrons and protons are the dominant component which contribute to the total energy density of the universe as non-relativistic particles i.e. $\rho_m \approx \rho_e + \rho_p$. As the number density of electron and protons are the same, as has been considered earlier, and the protons are much heavier than the electrons, one can consider that the non-relativistic fluid of the cosmic plasma is dominated by the proton fluid ($\rho_M \approx \rho_p$). Following such arguments one can ignore the electron contributions in Eq. (35) and write

$$K_{\text{CSL, RD}} \approx \sum_{s=e,p} \frac{3 \times 10^{-9} \lambda \hbar^2 \alpha}{8 m_s m_p c^2 \Omega_{R_0}^{\frac{1}{2}} H_0}. \quad (38)$$

Thus to ensure that the standard evolution of the non-relativistic plasma during RD era is not disturbed due to CSL heating one requires

$$K_{\text{CSL, RD}} \left[\frac{1}{(1 + z_f)^2} - \frac{1}{(1 + z_i)^2} \right] \ll 1, \quad (39)$$

where $z_i \approx 2 \times 10^9$ and $z_f \approx 3233$.

Thus in the generic scenario of CSL model (where the total change in energy density will depend upon the change in the electrons' energy density) one obtains the constraint on λ using Eq. (22) as

$$\lambda|_{\text{RD}} \ll 5 \times 10^{10} \text{ s}^{-1}, \quad (40)$$

while for the variant scenario with λ quadratically proportional to the mass one puts an upper bound on the parameter using Eq. (23) as

$$\lambda_0|_{\text{RD}} \ll 10^{14} \text{ s}^{-1}. \quad (41)$$

During MD era using Eq. (33) in Eq. (30) one gets

$$\frac{d\rho_M(z)}{\rho_M(z)} = 3 \frac{dz}{1 + z} - \sum_{s=e,p} \frac{f \lambda \hbar^2 \alpha}{4 m_s (m_e + m_p) c^2 H_0 \Omega_{M_0}^{\frac{1}{2}}} \frac{dz}{(1 + z)^{\frac{5}{2}}}, \quad (42)$$

and solving which yields

$$\rho_{M_f} = \exp \left\{ K_{\text{CSL,MD}} \left[\frac{1}{(1+z_f)^{\frac{3}{2}}} - \frac{1}{(1+z_i)^{\frac{3}{2}}} \right] \right\} \left(\frac{1+z_f}{1+z_i} \right)^3 \rho_{M_i}, \quad (43)$$

where we have defined

$$K_{\text{CSL,MD}} \equiv \sum_{s=e,p} \frac{f \lambda \hbar^2 \alpha}{6 m_s (m_e + m_p) c^2 \Omega_{M_0}^{\frac{1}{2}} H_0}. \quad (44)$$

But the situation changes in the MD era i.e. just before and after the recombination. Here we will consider the redshift span of $3233 < z < 1$. The recombination happens at $z \approx 1100$ which implies that for $z < 1100$ the matter and radiation will cease to remain tightly coupled and thereafter they will hardly interact. Thus, nearly all the heat gained by the non-relativistic matter due to CSL heating after $z \approx 1100$ will be retained in the matter itself, yielding $f \approx 1$. Thus the strongest bound on λ will come from the era $1100 < z < 1$. Also after recombination the energy density would be dominated by neutral hydrogen ($\rho_M \approx \rho_H$). Taking $\rho_H \approx \rho_p$ we can consider $\rho_M \approx \rho_p$ throughout $1100 < z < 1$. Following such arguments then one can drop the contribution of the electrons in the MD era as well and defining

$$K_{\text{CSL,MD}} \approx \sum_{s=e,p} \frac{\lambda \hbar^2 \alpha}{6 m_s m_p c^2 \Omega_{M_0}^{\frac{1}{2}} H_0}, \quad (45)$$

one should have

$$K_{\text{CSL,MD}} \left[\frac{1}{(1+z_f)^{\frac{3}{2}}} - \frac{1}{(1+z_i)^{\frac{3}{2}}} \right] \ll 1, \quad (46)$$

to ensure that the non-relativistic fluid evolves according to the standard cosmology during the MD era. Taking $z_i \approx 1100$ and $z_f \approx 1$ for the MD era the above condition puts an upper bound on the CSL parameter λ for the generic case as

$$\lambda|_{\text{MD}} \ll 4 \times 10^{-4} \text{ s}^{-1}, \quad (47)$$

where we have used $\Omega_{M_0} \approx 0.04$ as the present baryon density parameter. Similarly for the mass-dependent case one gets the upper bound as

$$\lambda_0|_{\text{MD}} \ll 0.7 \text{ s}^{-1}. \quad (48)$$

4. BOUNDS ON λ FROM μ -TYPE DISTORTION OF CMB

The thermalization history of the CMBR photons is affected by any unusual energy injection during its period of thermalization with the baryons in cosmic soup and results in spectral distortions of the observed CMBR blackbody spectrum [14, 17]. The unusual energy release in these early epochs can be due to decay of relic particles [18], by evaporation of primordial black holes [19], WIMP annihilation [20], damping of sound waves in primordial plasma [21] or by other astrophysical mechanisms as has been mentioned in [20]. Thermalization of the photons requires non-conservation of photon number in the cosmic soup. Cosmologically double-Compton scattering and bremsstrahlung processes are the ones which produce photons and thus are very significant in the thermalization of photons in the cosmic plasma. These photon number non-conserving processes are efficient in erasing possible spectral distortions in the early epochs. On the other hand, the photon-number conserving process of elastic Compton scattering, $\gamma + e^- \rightarrow \gamma + e^-$, is the most dominant process which couples electrons with photons at early times and redistributes the photons in frequency. As the photon number non-conserving processes, mainly the double Compton scattering process, become inefficient by $z \sim 10^6$ and photon-number conserving Compton scattering process then dominates the thermalization process, any energy release after $z \lesssim 10^6$ leads to spectral distortion of CMBR.

Energy releases during $2 \times 10^6 > z > 5 \times 10^4$ lead to a kind of CMBR spectral distortion, called μ -type spectral distortion, which yields a non-vanishing frequency dependent chemical potential of the CMBR photons. This epoch $2 \times 10^6 > z > 5 \times 10^4$ is known as the μ -era in the literature. Due to such energy releases the Planck spectrum of the thermalized photon density relaxes to a Bose-Einstein distribution with a non-zero chemical potential μ [14] as

$$n_{\text{Pl}} \equiv \frac{1}{e^{\epsilon/\kappa_B T} - 1} \quad \longrightarrow \quad n_{\text{BE}} \equiv \frac{1}{e^{(\epsilon+\mu)/\kappa_B T} - 1}. \quad (49)$$

Absence of significant distortion in the CMBR spectrum from the Black Body distribution observed by COBE/FIRAS puts an upper limit on such distortion as $\mu \lesssim 9 \times 10^{-5}$ [15]. An upcoming space-based mission, called PIXIE, will constrain the μ -type distortion up to $\mu \sim 5 \times 10^{-8}$ [16].

An interesting feature of these processes is that while double-Compton scattering and bremsstrahlung processes become efficient as frequency decreases, elastic Compton scat-

tering process, on the other hand, is independent of the frequency of the photons. Thus at low redshifts when the Compton scattering is the dominant process of thermalization of the cosmic plasma, the photon number non-conserving double-Compton scattering and bremsstrahlung processes are still efficient at low frequencies and are capable of retaining the black body spectrum [22]. Also, the photons, which are generating at low frequencies and getting scattered to high frequency regime, are not sufficient to maintain a Planck spectrum at high frequencies. Thus the elastic Compton scattering establishes a Bose-Einstein spectrum only at the high frequencies of the spectrum.

The total μ -distortion generated due to energy injections and injection of high-energy photons during $2 \times 10^6 > z > 5 \times 10^4$ can be written as as [22]

$$\delta\mu = \frac{1}{2.143} \left(3 \frac{\delta\varepsilon}{\varepsilon} - 4 \frac{\delta n}{n} \right), \quad (50)$$

where $\delta\varepsilon$ and δn are the total changes in energy and number density respectively with respect to a Planckian distribution. A brief derivation of the above formula is given in Appendix (A). This remarkable result shows that the total energy distortion of μ -type is independent of the actual form of the energy injection. It can also be seen from the above equation that direct heating of the electrons contributes in the same way as injection of high-frequency photons as has been pointed out in [22].

It is important to note here that Compton scattering during these epochs is very efficient to maintain the equilibrium between the electron and photon temperature and keeps the cosmic fluid tightly coupled. Thus the CSL heating rate should be much smaller than the Compton scattering rate such that the thermal equilibrium of the cosmic plasma should be retained. During these epochs equilibrium is attained within a time scale given as [22]

$$t_C = \frac{3m_e c}{4\sigma_T \varepsilon_\gamma} \simeq 7.63 \times 10^{19} (1+z)^{-4} \text{ s}, \quad (51)$$

where ε_γ is the energy density of photons at that redshift. Also, in the above equation the scattering of photons with electrons is only considered as that with protons is 10^6 times slower and $e^- + \gamma \rightarrow e^- + \gamma$ would be the dominant process to keep the plasma in thermal equilibrium. Thus one would expect the CSL heating rate to be much smaller than the rate of attaining equilibrium (t_C^{-1}) via Compton scattering till the recombination ends at $z \sim 1100$, which using Eq. (50) weakly puts an upper bound on the CSL heating rate for

the generic case as

$$\lambda|_{\text{CS}} < \frac{4m_e^2 c^2}{3\alpha\hbar^2} t_C^{-1} \Big|_{z \sim 1100} \approx 2 \times 10^3 \text{ s}^{-1}, \quad (52)$$

where we have used the values $m_e \approx 9.1 \times 10^{-31} \text{ kg}$, $\alpha \approx 10^{14} \text{ m}^{-2}$, $c = 3 \times 10^8 \text{ m s}^{-1}$ and $\hbar = 6.62 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$. Similar bounds in the variant scenario where λ is quadratically mass-proportional would be

$$\lambda_0|_{\text{CS}} < \frac{4m_p m_e c^2}{3\alpha\hbar^2} t_C^{-1} \Big|_{z \sim 1100} \approx 3 \times 10^6 \text{ s}^{-1}. \quad (53)$$

In a generic scenario of CSL model [1, 4], non-relativistic particles gain energy with a rate λ without changing the photon number density at higher frequencies. Here we will assume that the amount of energy gained by the electrons or protons due to CSL heating will be transferred to the photons completely as during these epochs the number density of photons exceeds that of the electrons by many orders of magnitude. Thus the total μ -distortion due to CSL heating will be

$$\delta\mu = \frac{3}{2.143} \frac{\delta\varepsilon_{\text{CSL}}}{\varepsilon_\gamma}, \quad (54)$$

where we have neglected the last term in Eq. (50) as there is no significant photon production in a generic CSL scenario at high-frequencies. During the μ -era the total amount of energy gained by the non-relativistic fluid in the cosmic plasma can be determined by using Eq. (21). Using Eq. (29) in Eq. (21) yields the rate of energy gain by the non-relativistic particles with redshift as

$$\frac{\delta\varepsilon}{\delta z} = - \sum_{s=e,p} \frac{3\alpha\lambda\hbar^2}{4m_s} n_{e0} \frac{(1+z)^2}{H(z)}, \quad (55)$$

where we have used the relation between the present number density of electrons n_{e0} and that in a particular redshift $n_e(z)$ as $n_e(z) = n_{e0}(1+z)^3$. During the radiation era, i.e. before recombination, the Hubble parameter $H(z)$ can be written as $H(z) \simeq \Omega_{R0}^{\frac{1}{2}} H_0 (1+z)^2$ where $\Omega_{R0} \approx 6.5 \times 10^{-5}$ and $H_0 \approx 2 \times 10^{-18} \text{ s}^{-1}$ are the present radiation density parameter and Hubble parameter respectively. Putting this value for $H(z)$ in the above equation yields

$$\frac{\delta\varepsilon}{\delta z} = - \sum_{s=e,p} \frac{3\alpha\lambda\hbar^2}{4m_s} \frac{n_{e0}}{\Omega_{R0}^{\frac{1}{2}} H_0}. \quad (56)$$

The unperturbed Planckian photon energy at z would be

$$\varepsilon(z) = \frac{4\sigma_B T_0^4}{c} (1+z)^4, \quad (57)$$

which will give the fractional change in the energy of a pure Planckian spectrum at redshift z as

$$\frac{1}{\varepsilon} \frac{\delta\varepsilon}{\delta z} = - \sum_{s=e,p} \frac{3\alpha\lambda\hbar^2}{16m_s} \frac{n_{e_0}c}{\Omega_{R_0}^{\frac{1}{2}} H_0 \sigma_B T_0^4} (1+z)^{-4}, \quad (58)$$

where $T_0 \sim 2.73$ K is the present temperature of the CMBR. Integrating the above equation over $z_i = 2 \times 10^6$ to $z_f = 5 \times 10^4$ one gets the total fractional energy gained by the electron or proton plasma due to CSL heating during the μ -era as

$$\frac{\Delta\varepsilon}{\varepsilon} = \sum_{s=e,p} \frac{\alpha\lambda\hbar^2}{16m_s} \frac{n_{e_0}c}{\Omega_{R_0}^{\frac{1}{2}} H_0 \sigma_B T_0^4} \left[\frac{1}{(1+z_f)^3} - \frac{1}{(1+z_i)^3} \right]. \quad (59)$$

In a generic case, where the CSL parameter λ is independent of mass, we have shown that the gain in the electron energy density will dominate and this will also be the total energy gained by the photons via Compton scattering through electrons. Hence the total μ -distortion generated due to CSL heating will be

$$\mu \simeq \frac{3}{2.143} \times \frac{\alpha\lambda\hbar^2}{16m_e} \frac{n_{e_0}c}{\Omega_{R_0}^{\frac{1}{2}} H_0 \sigma_B T_0^4} \left[\frac{1}{(1+z_f)^3} - \frac{1}{(1+z_i)^3} \right] \sim 1.2 \times 10^{-6} \lambda \text{ s}, \quad (60)$$

where apart from the previously mentioned values of the parameters we have used $n_{e_0} \approx 0.246 \text{ m}^{-3}$. Using the COBE/FIRAS measurement of μ -distortion in the CMBR spectrum [15] one then can put an upper bound on the CSL heating parameter as

$$\lambda|_{\mu, \text{COBE/FIRAS}} \lesssim 70 \text{ s}^{-1}, \quad (61)$$

whereas upcoming experiment PIXIE [16] can put a more stringent constraint on the CSL heating parameter as

$$\lambda|_{\mu, \text{PIXIE}} \lesssim 4 \times 10^{-2} \text{ s}^{-1}. \quad (62)$$

Similarly for the case where λ is quadratically mass-proportional, photons will gain energy from both the electrons and the protons via Compton scattering. Hence, the total μ -distortion can be calculated using Eq. (23) and Eq. (57) as

$$\mu \simeq \frac{3}{2.143} \times \frac{\alpha\lambda_0\hbar^2}{16m_p} \times \frac{n_{e_0}c}{\Omega_{R_0}^{\frac{1}{2}} H_0 \sigma_B T_0^4} \left[\frac{1}{(1+z_f)^3} - \frac{1}{(1+z_i)^3} \right] \sim 6.8 \times 10^{-10} \lambda_0 \text{ s}. \quad (63)$$

Thus bound on λ_0 from observations of COBE/FIRAS would be

$$\lambda_0|_{\mu, \text{COBE/FIRAS}} \lesssim 10^5 \text{ s}^{-1}, \quad (64)$$

and that of from PIXIE would be

$$\lambda_0|_{\mu, \text{PIXIE}} \lesssim 74 \text{ s}^{-1}. \quad (65)$$

5. BOUNDS ON λ FROM y -TYPE DISTORTION OF CMB

Energy release in the early universe during $z \lesssim 5 \times 10^4$ leads to a y -type CMBR spectral distortion [17]. At these low redshifts Compton scattering is not effective enough to maintain the full kinetic equilibrium between photons and electrons. Due to energy releases if the temperature of the electrons becomes greater than that of the photons then the low-energy photons will be upscattered in frequency via Compton scattering. Thus a y -type distortion is characterized by a deficit of photons in the low frequency regime with an increment of photons at high frequencies in comparison with a standard blackbody spectrum. The efficiency of Compton scattering process to maintain a blackbody spectrum is quantified by a parameter, called the Compton y -parameter, as

$$y = \int \frac{\kappa_b(T_e - T_\gamma)}{m_e c^2} N_e \sigma_T c dt, \quad (66)$$

where κ_b is the Boltzmann constant, T_e and T_γ are the temperatures of the electrons and photons in the cosmic plasma respectively and N_e is the electron number density. The present constraint on such distortion of CMBR comes from COBE/FIRAS experiments which puts an upper bound on y -parameter as $y \leq 1.5 \times 10^{-5}$ with 95% confidence [15]. The upcoming experiment PIXIE can probe CMBR distortions up to $y \leq 10^{-8}$ [16].

Energy injection of $\delta\varepsilon_\gamma$ in the photon energy density during $z \lesssim 5 \times 10^4$ can yield a Compton y -parameter as [17, 23]

$$\delta y = \frac{1}{4} \frac{\delta\varepsilon}{\varepsilon}. \quad (67)$$

A brief derivation of the above formula is given in Appendix (B). In this work we are mainly interested in y -type distortions arising during the pre-recombinational era i.e. during $1100 < z < 5 \times 10^4$. This is because any energy injection occurring after the completion of hydrogen recombination would not imprint any additional traces in the CMBR distortions. Later on, in post-recombinational epochs ($z \lesssim 800$), different physical mechanisms, like interaction of CMBR with hot intergalactic gas known as thermal Sunyaev-Zeldovich effect [24]; supernova remnants at high redshifts or large-scale structure formation giving rise to shock waves [25, 26], can also contribute to the y -distortion observed in CMBR. Contribution to y -distortion coming from such effects have been ignored in this study.

As the dominant process during these epochs is still the elastic Compton scattering, we follow the same procedure to calculate the total energy gain by the photons, due to

CSL heating of non-relativistic electrons or protons present in the cosmic plasma, as we have discussed while calculating the μ -distortion. It is important to note here that during the y -era the redshift span $5 \times 10^4 < z < 3233$ is radiation dominated while the rest $3233 < z < 1100$ is matter dominated. Thus generalizing Eq. (59) and Eq. (57) to reflect this fact and using those in Eq. (67) in the redshift span $z_i = 5 \times 10^4$ to $z_f = 1100$ one computes the total y -distortion generated due to generic CSL heating as

$$y \simeq \frac{1}{4} \times \frac{3\alpha\lambda\hbar^2}{4m_e} \frac{n_{e0}c}{4\sigma_B T_0^4 H_0} \left[\frac{1}{3\Omega_{R0}^{\frac{1}{2}}} \left\{ \frac{1}{(1+z_f^R)^3} - \frac{1}{(1+z_i^R)^3} \right\} + \frac{5}{2\Omega_{M0}^{\frac{1}{2}}} \left\{ \frac{1}{(1+z_f^M)^{\frac{5}{2}}} - \frac{1}{(1+z_i^M)^{\frac{5}{2}}} \right\} \right] \sim 0.2 \lambda \text{ s}. \quad (68)$$

Here we have taken $z_i^R \approx 5 \times 10^4$ and $z_f^R \approx 3233$ for the radiation dominated epoch in y -era and similarly $z_i^M \approx 3233$ and $z_f^M \approx 1100$ for the matter dominated epoch. Thus the constraint on λ from the measurement of y -parameter of CMBR by COBE/FIRAS experiment [15] would be

$$\lambda|_{y,\text{COBE/FIRAS}} \lesssim 8 \times 10^{-5} \text{ s}^{-1}, \quad (69)$$

whereas in future measurement obtained by PIXIE [16] can put an upper bound on λ as

$$\lambda|_{y,\text{PIXIE}} \lesssim 5 \times 10^{-8} \text{ s}^{-1}. \quad (70)$$

Similarly for the variant model where the parameter λ is mass-dependent one has for the total y -distortion as

$$y \simeq \frac{1}{4} \times \frac{3\alpha\lambda_0\hbar^2}{4m_p} \frac{n_{e0}c}{4\sigma_B T_0^4 H_0} \left[\frac{1}{3\Omega_{R0}^{\frac{1}{2}}} \left\{ \frac{1}{(1+z_f^R)^3} - \frac{1}{(1+z_i^R)^3} \right\} + \frac{5}{2\Omega_{M0}^{\frac{1}{2}}} \left\{ \frac{1}{(1+z_f^M)^{\frac{5}{2}}} - \frac{1}{(1+z_i^M)^{\frac{5}{2}}} \right\} \right] \sim 10^{-4} \lambda \text{ s}. \quad (71)$$

Thus the upper bound on λ_0 obtained from COBE/FIRAS observations would be

$$\lambda_0|_{y,\text{COBE/FIRAS}} \lesssim 0.14 \text{ s}^{-1}, \quad (72)$$

and that coming from PIXIE's upcoming observations

$$\lambda_0|_{y,\text{PIXIE}} \lesssim 10^{-4} \text{ s}^{-1}. \quad (73)$$

6. BRIEF DISCUSSION ON CSL MODEL WITH SPONTANEOUS PHOTON EMISSION AND ITS COSMOLOGICAL IMPLICATIONS

In the literature it has been shown that for collapse models, atomic systems [13, 27, 28] or free electrons [29] emit radiations spontaneously when the noise term in the evolution is treated perturbatively. The rate of creation of photons with wavenumber k due to spontaneous collapse of a (free) particle species s can be calculated as

$$\frac{d\Gamma(k)}{dk} \sim \frac{\lambda \alpha \hbar^2 e^2}{2\pi^2 \epsilon_0 c^3 m_s^2 k}. \quad (74)$$

The change in the number density n_γ of photons due to this process would be

$$\left[\frac{dn_\gamma}{dt} \right]_{\text{CSL}} \sim \frac{\lambda \alpha \hbar^2 e^2 n_{s0} (1+z)^3 \log k}{2\pi^2 \epsilon_0 c^3 m_s^2} \approx 10^{-48} \log(k) \lambda (1+z)^3, \quad (75)$$

where s has been taken as electrons. This injection of photons into the cosmic plasma will also contribute to the spectral distortion of CMBR in principle, as can be seen from Eq. (50). However, we can see from the above equation that the injection of photons due to spontaneous collapse will not provide any significant contribution at higher frequencies and will be suppressed by n as well. Even at lower frequencies this process is clearly subdominant compared to other photon producing processes like bremsstrahlung and double Compton scattering where the rates of photon creation are [22]

$$\left[\frac{dn_\gamma}{dt} \right]_{\text{br}} \sim \frac{10^{-18} \log(2.25/x_e)}{k^3 e^{x_e}} [1 - n_\gamma(e^{x_e} - 1)] z^{5/2} \quad (76)$$

and

$$\left[\frac{dn_\gamma}{dt} \right]_{\text{DC}} \sim \frac{10^{-32}}{k^3} I(t) [1 - n_\gamma(e^{x_e} - 1)] z^5, \quad (77)$$

respectively, for small x_e . In the above equations

$$x_e = \frac{hck}{k_B T_e}, \quad (78)$$

with T_e being the electron temperature and

$$I(t) = \int dx_e x_e^4 n_\gamma (n_\gamma + 1). \quad (79)$$

We can see from above that for small x_e and large n_γ

$$\frac{\left[\frac{dn_\gamma}{dt} \right]_{\text{CSL}}}{\left[\frac{dn_\gamma}{dt} \right]_{\text{br}}} = \frac{10^{-30} k^3 \log(k) e^{x_e}}{\log(2.25/x_e) [1 - n_\gamma(e^{x_e} - 1)]} z^{1/2} \longrightarrow 0, \quad (80)$$

and also

$$\frac{\left[\frac{dn_\gamma}{dt}\right]_{\text{CSL}}}{\left[\frac{dn_\gamma}{dt}\right]_{\text{DC}}} \longrightarrow 0. \quad (81)$$

Thus such a process at low frequencies are subdominant to other photon creating processes like double Compton scattering and bremsstrahlung which provide sufficient number of low energy photons to establish blackbody spectrum at lower frequencies. However, these photon creating processes including the one of spontaneous collapse model become more and more inefficient as frequency of the generated photons increases. Also the strength parameter λ of the collapse model is stringently constrained to very low values (as low as 10^{-5}) from present observations of CMBR spectral distortions. These arguments along with the fact that the spectral distortion is proportional to the number of photons generated which is suppressed by the total number of photons (the factor $\frac{\delta n}{n}$ of Eq. (50)) indicate that the generated photons at high-frequencies will not be able to yield significant distortions in the CMBR spectrum. Also $\frac{\delta n}{n}$ in the above equation can be significant if one takes λ to be large which then violates the perturbative analysis of the model. So such a photon emitting collapse process will hardly improve the bounds on the strength parameter λ obtained from μ -type and y -type distortions.

7. DISCUSSION AND CONCLUSION

In the present article we have addressed the effects of CSL heating of non-relativistic particles on standard cosmology and formation of CMBR. As the thermal evolution of our universe has been studied and understood rigorously by both theoretical and experimental means during the past half a century, any anomalous heating, which can disturb the thermal evolution of our universe, thus can be constrained stringently by observations. Motivated by this, we endeavor to investigate the effects a model like CSL, which heats the non-relativistic particles with a constant rate, can have on cosmological evolution and on CMBR formation. In the following Table I we summarize the bounds we obtain by ensuring that the CSL heating would not hamper the standard cosmological evolution of the non-relativistic particles and also it would not generate substantial distortions (μ -type and y -type) in the observed precise blackbody spectrum of CMBR. It is to be noted from Table I that the strongest bounds are coming from the observations of y -distortion in the CMBR. Such

TABLE I: Bounds on CSL strength parameter

Case	λ (in s^{-1})	λ_0 (in s^{-1})
Bounds from RD era of standard cosmology	$\ll 5 \times 10^{10}$	$\ll 10^{14}$
Bounds from MD era of standard cosmology	$\ll 4 \times 10^{-4}$	$\ll 0.7$
Bounds from comparing rates of Compton scattering and CSL heating	2×10^3	3×10^6
Bounds from COBE/FIRAS observation of μ -distortion	70	10^5
Bounds from PIXIE future observation of μ -distortion	4×10^{-2}	74
Bounds from COBE/FIRAS observation of y -distortion	8×10^{-5}	0.14
Bounds from PIXIE future observation of y -distortion	5×10^{-8}	10^{-4}

a bound on the strength parameter λ or λ_0 is within the detection range of the proposed diffraction experiments from fullerene and larger molecules [2, 12]. It suggests that apart from laboratory experiments, cosmological scenarios are also very important to test any consistent collapse model which is in accordance with the standard FRW framework of cosmology.

In [13] a stronger bound on the strength parameter λ_0 has been obtained from the observed thermal equilibrium of IGM in the redshift span $z \sim 6 - 2$, by comparing the rate of heating of IGM through CSL mechanism with the adiabatic cooling of the same due to Hubble expansion. The slow cooling rate due to Hubble expansion ensures that any heating mechanism during that period should also be very small, constraining the parameter of the model to a much smaller value (as small as 10^{-8} s^{-1}). While Adler's analysis considers bounds yielding due to maintaining thermal equilibrium of the IGM, the analysis done in this article studies the breakdown of thermal equilibrium between photons and matter in cosmic plasma before recombination due to CSL heating and thus constrains the energy intake by the photons in the cosmic plasma before $z > 1100$. As the processes, like Compton scattering, are very efficient in redistributing the energy over the spectrum and thus thermalizing any anomalous heat-injection, a large enough heat injected through CSL can be tolerated by the cosmic soup without generating large spectral distortions and thus yielding a weaker bound on the strength parameter than the one obtained from IGM heating [13]. On the other hand, while calculating the heat intake by the CMBR photons due to heating of protons by CSL mechanism over the age of the universe, the case which is similar to the

ones considered in this article, Adler too found a bound of $\lambda_0 < 10^{-5} \text{ s}^{-1}$ [13] similar to the ones obtained here.

It is nevertheless very important to mention at this point that the above bounds have been obtained by considering only CSL effect as the sole energy injecting process before recombination. However there can be other mechanisms which can lead to injection of energies at such high-redshifts like adiabatic cooling of ordinary matter [30, 31], evaporating black holes [19], decaying particles [18, 22], dissipation of magnetic fields [32] or superconducting strings [33]. Some of these processes are theoretically well understood and thus the distortion associated with such mechanisms can be obtained precisely. Hence the bounds on the strength parameter λ obtained in this article are the most conservative ones. Any other energy-release mechanism can further tighten the bounds obtained here. Furthermore, in this article only white noise field has been considered for the stochastic field $W_t(\mathbf{x})$ required for the collapse processes in CSL model. There have been attempts motivated from energy conservation to model CSL mechanism with non-white noise [34–36], which is primarily dominant in low frequency regime. It could be interesting to analyze this kind of collapse mechanism in the cosmological context and can be ventured in future studies.

Appendix A: Brief derivations of μ -type distortion

Here we will give a brief derivation of μ -distortion following [14, 22]. For a small chemical potential ($\mu \ll 1$) the energy density of a Bose-Einstein distribution is given as

$$\varepsilon = \frac{4\sigma_B T^4}{c} \left(1 - 3 \frac{I_2}{I_3} \mu \right), \quad (\text{A1})$$

where $\sigma_B \approx 5.67 \times 10^{-8} \text{ kg s}^{-3} \text{ K}^{-4}$ is the Stefan-Boltzmann constant. The number density of photons in this distribution is

$$n = \frac{4\sigma_B T^3}{kc} \left(1 - 2 \frac{I_1}{I_2} \mu \right). \quad (\text{A2})$$

The constants I_n are related to the Reimann ζ function as

$$I_n = \int_0^\infty dx \frac{x^n}{e^x - 1} = n! \zeta(n+1), \quad (\text{A3})$$

with the numerical values $I_1 \simeq 1.645$, $I_2 \simeq 2.404$ and $I_3 \simeq 6.494$. Energy release processes in the early epochs may involve direct energy injection or injection of high frequency photons.

The change in the energy density given in Eq. (A1) is given by

$$\frac{\delta\varepsilon}{\varepsilon} = 4\delta\ln T - 3\frac{I_2}{I_3}\delta\mu, \quad (\text{A4})$$

whereas the change in the number density given in Eq. (A2) will be

$$\frac{\delta n}{n} = 3\delta\ln T - 2\frac{I_1}{I_2}\delta\mu. \quad (\text{A5})$$

Solving the above two equations simultaneously one obtains the change in chemical potential of the photons in the Bose-Einstein distribution as given in Eq. (50).

Appendix B: Brief derivations of y -type distortion

Here we will give a brief derivation to calculate the y -parameter generated due to energy injection. In the non-relativistic limit, described by $\kappa_b T_e \ll m_e c^2$ and $\kappa_b T_\gamma \ll m_e c^2$, Kompaneets' equation is the kinetic equation of Compton scattering which can be expressed as [22]

$$\frac{\partial n_\gamma}{\partial t} = N_e \sigma_{Tc} \left(\frac{\kappa_b T_e}{m_e c^2} \right) \frac{1}{x_e^2} \frac{\partial}{\partial x_e} \left[x_e^4 \left(\frac{\partial n_\gamma}{\partial x_e} + n_\gamma + n_\gamma^2 \right) \right], \quad (\text{B1})$$

where n_γ is the photon occupation number and $x_e \equiv \frac{h\nu}{\kappa_b T_e}$. The above equation simplifies when T_e is much larger, yielding [17]

$$\frac{\partial n_\gamma}{\partial t} = N_e \sigma_{Tc} \left(\frac{\kappa_b T_e}{m_e c^2} \right) \frac{1}{x_e^2} \frac{\partial}{\partial x_e} \left(x_e^4 \frac{\partial n_\gamma}{\partial x_e} \right). \quad (\text{B2})$$

Using the definition of Compton y -parameter given in Eq. (66) the above equation can be written as

$$\frac{\partial n_\gamma}{\partial y} = \frac{1}{x_e^2} \frac{\partial}{\partial x_e} \left(x_e^4 \frac{\partial n_\gamma}{\partial x_e} \right), \quad (\text{B3})$$

where we have dropped the term T_γ in Eq. (66) as in this case $T_e > T_\gamma$. The energy density ε_γ of photons can be written in terms of occupation number n_γ as

$$\varepsilon_\gamma = \frac{8\pi h}{c^3} \int n_\gamma \nu^3 d\nu = \frac{8\pi}{h^3 c^3} (\kappa_b T_e)^4 \int n_\gamma x_e^3 dx_e. \quad (\text{B4})$$

Thus multiplying both sides of Eq. (B3) by x_e^3 and solving for the integration yields

$$\frac{\delta\varepsilon_\gamma}{\delta y} = 4\varepsilon_\gamma. \quad (\text{B5})$$

Thus energy injection of $\delta\varepsilon_\gamma$ during $z \lesssim 5 \times 10^4$ can yield a Compton y -parameter as given in Eq. (67).

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